Quantum Chromodynamics

& LHC Phenomenology

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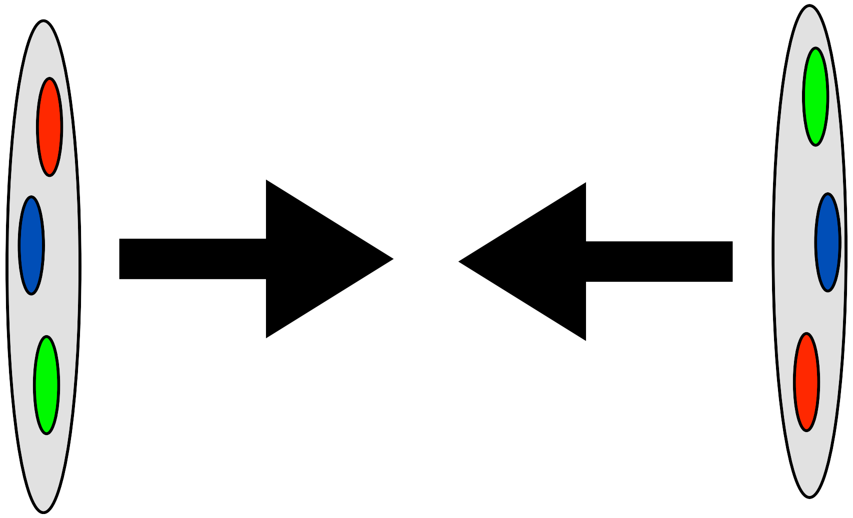
Partons in the initial state

- We talked a lot about final state QCD effects
- This is the only thing to worry about at e^+e^- colliders (LEP)
- Hera/Tevatron/LHC involve protons in the initial state
- Proton are made of QCD constituents

This lecture will focus mainly on aspects related to initial state effects

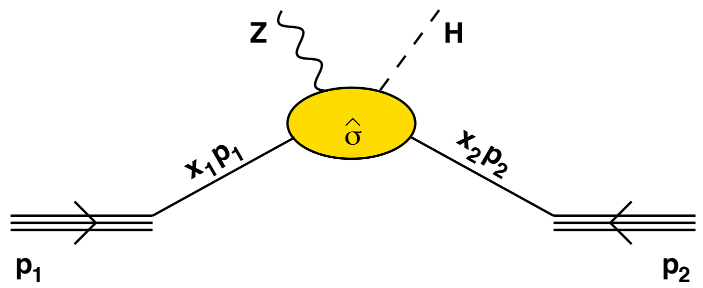


The parton model

Basic idea of the parton model: intuitive picture where in a high transverse momentum scattering partons behave as quasi free in the collision

$\Rightarrow$ cross section is the incoherent sum of all partonic cross-sections

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \qquad \hat{s} = x_1 x_2 s$$



NB: This formula is wrong/incomplete (see later)

$f_i^{(P_j)}(x_i)$: parton distribution function (PDF) is the probability to find parton i in hadron j with a fraction x_i of the longitudinal momentum (transverse momentum neglected), extracted from data

$\hat{\sigma}(x_1 x_2 s)$: partonic cross-section for a given scattering process, computed in perturbative QCD

Sum rules

Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x_i f_i^{(p)}(x) = 1$$

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Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \sum_i \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x)\right) = 2$$

$$\int_0^1 dx \sum_i \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x)\right) = 1$$

$$\int_0^1 dx \sum_i \left(f_s^{(p)}(x) - f_{\bar{s}}^{(p)}(x)\right) = 0$$

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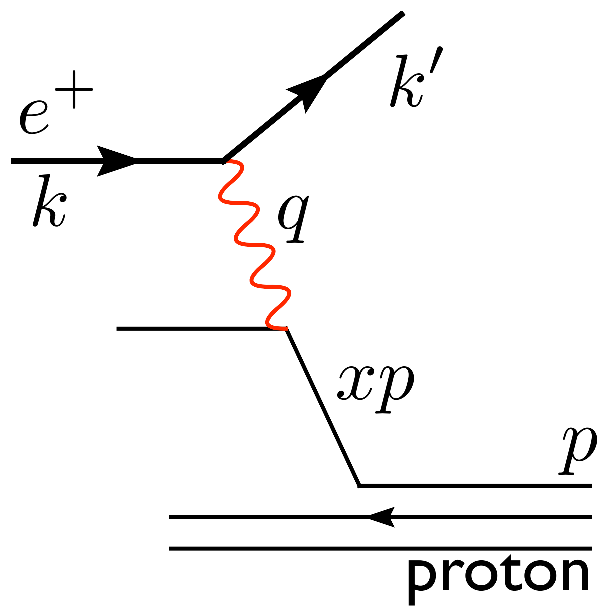
How can parton densities be extracted from data?

Deep inelastic scattering

Easier than processes with two incoming hadrons is the scattering of a lepton on a (anti)-proton



Kinematics:

$$Q^2 = -q^2 \quad s = (k+p)^2 \quad x_{Bj} = \frac{Q^2}{2p\cdot q} \quad y = \frac{p\cdot q}{k\cdot p}$$

Partonic variables:

$$\hat{p} = xp \quad \hat{s} = (k+\hat{p})^2 = 2k\cdot\hat{p} \quad \hat{y} = \frac{\hat{p}\cdot q}{k\cdot\hat{p}} = y \quad (\hat{p}+q)^2 = 2\hat{p}\cdot q - Q^2 = 0$$

Partonic cross section: (just QED Feynman rules)

$$\boxed{\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\,\pi\,\alpha_{em}\left(1+(1-\hat{y})^2\right)}$$

Deep inelastic scattering

<u>Hadronic cross section:</u>

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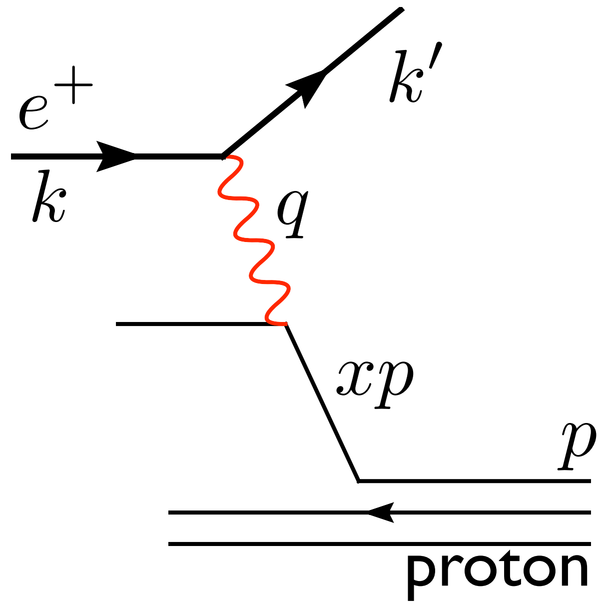

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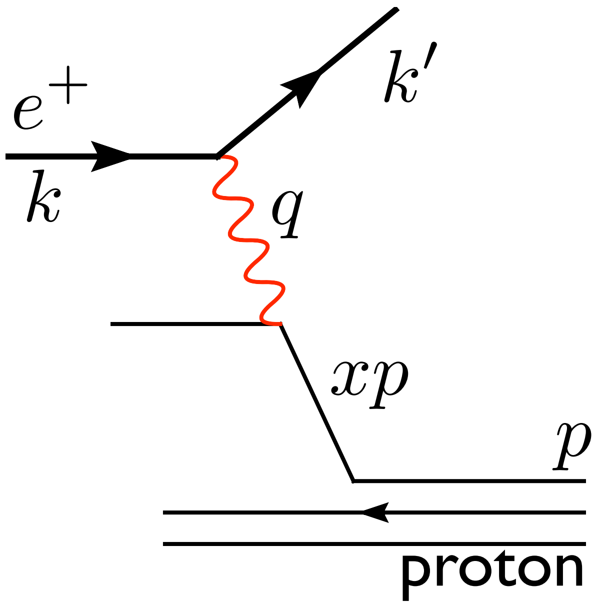

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1. at fixed x_{Bj} and y the cross-section scales with s
2. the y-dependence of the cross-section is fully predicted and is typical of vector interaction with fermions (Callan-Gross relation)
3. can access parton distribution functions
4. Bjorken scaling: pdfs depend on x and not on Q^2

Exercise: show that in the CM frame of the electron-quark system y is given by $(1-\cos\theta_{\mathrm{el}})/2$, with θ_{el} the scattering angle of the electron in this frame

Exercise:

- show that the two particle phase space is $\dfrac{d\phi}{16\pi}$
- show that the squared matrix element is $\dfrac{16\pi\alpha q_l^2}{Q^4}\hat{s}xpk\left(1+(1-y)^2\right)$
- show that the flux factor is $\dfrac{1}{4xpk}$

Hence derive that

$$\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\,\pi\,\alpha_{em}\left(1+(1-\hat{y})^2\right)$$

Definition: F_2

$$\frac{d\sigma}{dy dx} = \frac{2\pi\alpha_{em}^2 s}{Q^4}\left(1+(1-y^2)\right)F_2(x) \qquad F_2(x) = \sum_l x q_l^2 f_l^{(p)}(x)$$

F_2 is called structure function

For electron scattering on proton

$$F_2(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

NB: used perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Question: F_2 gives only a linear combination of u and d. How can they be extracted separately?

Isospin

Neutron is like a proton with u & d exchanged

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For electron scattering on a neutron

$$F_2^n(x) = x\left(\frac{1}{9}d_n(x) + \frac{4}{9}u_n(x)\right) = x\left(\frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)\right)$$

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F_2^n and F_2^p allow determination of u_p and d_p separately

NB: experimentally get F_2^n from deuteron: $F_2^d(x) = \frac{1}{2}\left(F_2^p(x) + F_2^n(x)\right)$

Sea quark distributions

Inside the proton there are fluctuations, and pairs of $u\bar{u}$,$d\bar{d}$,$c\bar{c}$,$s\bar{s}$ ... can be created

An infinite number of pairs can be created as long as they have very low momentum (because of momentum sum rule)

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx\,(u_p(x) - \bar{u}_p(x)) = 2 \qquad \int_0^1 dx\,\left(d_p(x) - \bar{d}_p(x)\right) = 1$$

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Photons interact in the same way with u(d) and $\bar{u}$($\bar{d}$)

How can one measure the difference?

Question: What interacts differently with particle and antiparticle?

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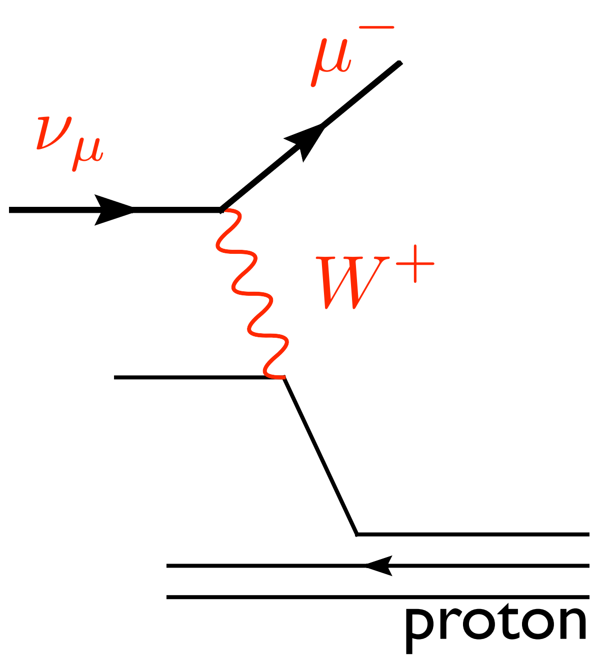
Question: What interacts differently with particle and antiparticle? W^+/W^- from neutrino scattering



Check of the momentum sum rule

$$\int_0^1 dx \sum_i x_i f_i^{(p)}(x) = 1$$

u_v	0.267
d_v	0.111
u_s	0.066
d_s	0.053
s_s	0.033
c_c	0.016
total	0.546

➟ *half of the longitudinal momentum is missing!*

Who is missing?

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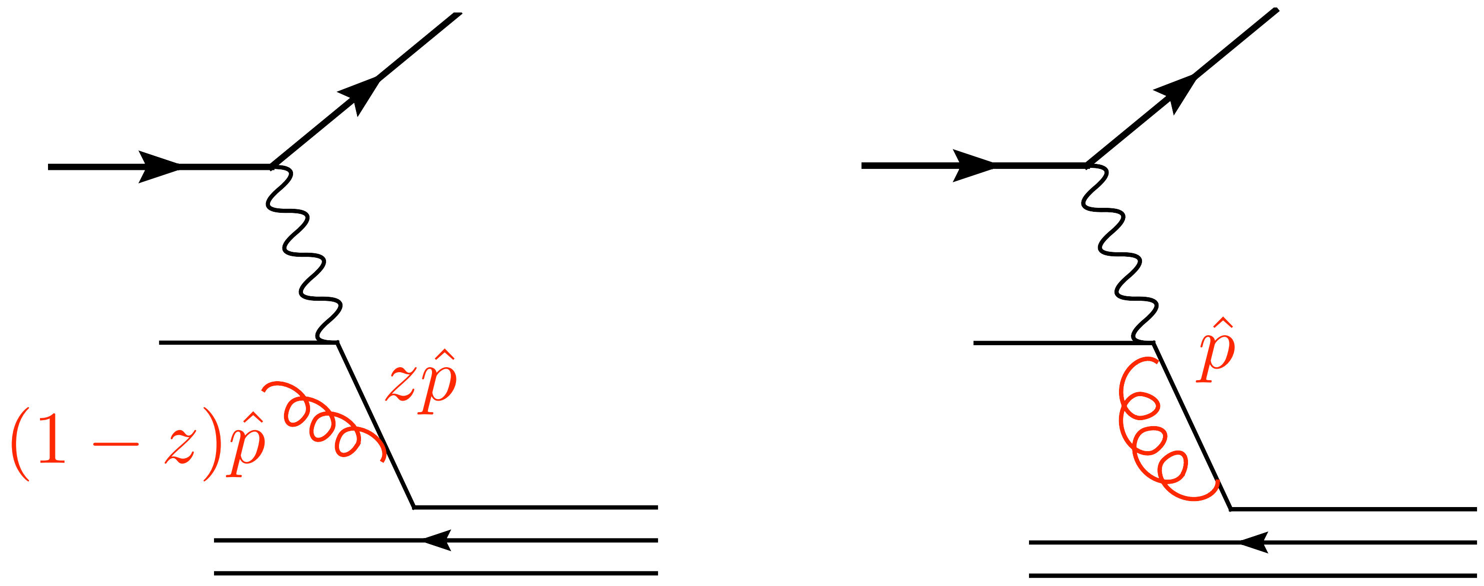
Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



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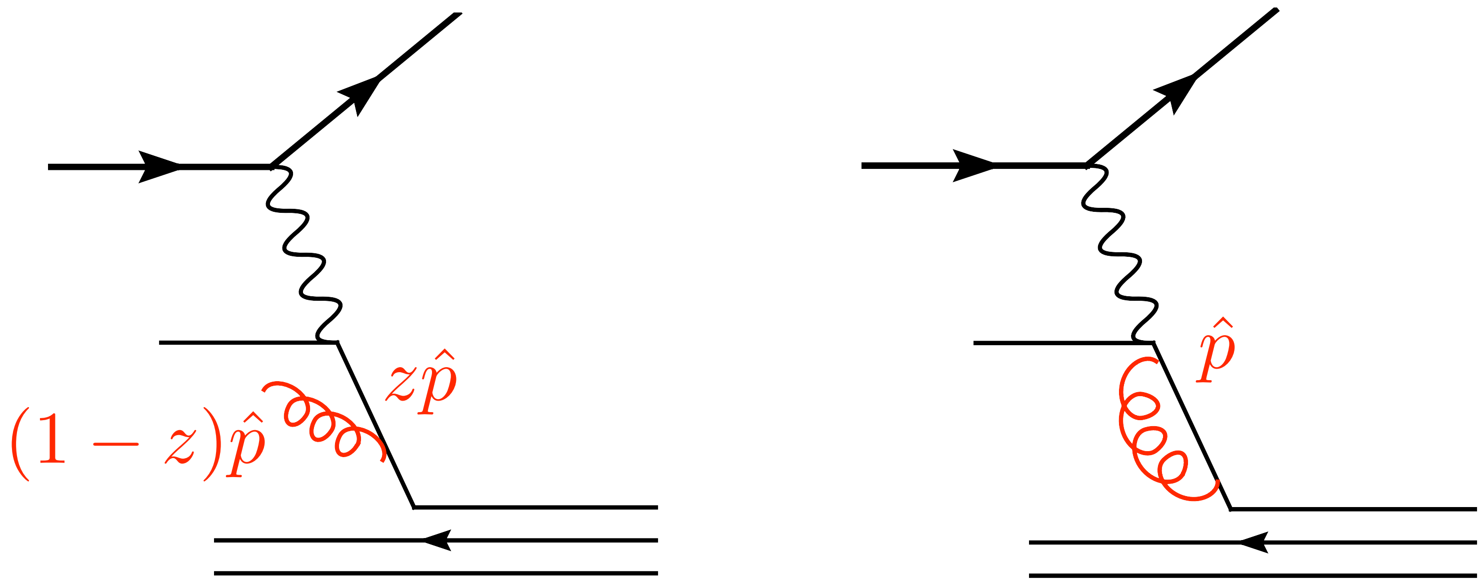


Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dl_\perp^2}{l_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $l_\perp \to 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

$\Longrightarrow$ naive parton model does not survive radiative corrections!

Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Plus prescription makes the universal cancelation of soft singularities explicit

$$\int_0^1 dz f(z)_+ g(z) = \int_0^1 dz f(z) \left(g(z) - g(1) \right)$$

The partonic cross section becomes

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z)_+ \sigma^{(0)}(z\hat{p}) \qquad P(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

Collinear singularities still there, but they factorize.

Factorization scale

Schematically use

$$\ln \frac{Q}{\lambda^2} = \ln \frac{Q}{\mu_F^2} + \ln \frac{\mu_F}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+\right) \sigma^{(0)}$$

So we define

$$f_q(x, \mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p, \mu_F) = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_{qq}^{(0)}\right) \sigma^{(0)}(p)$$

NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently

Intermediate recap

- Initial state emissions with $l_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders

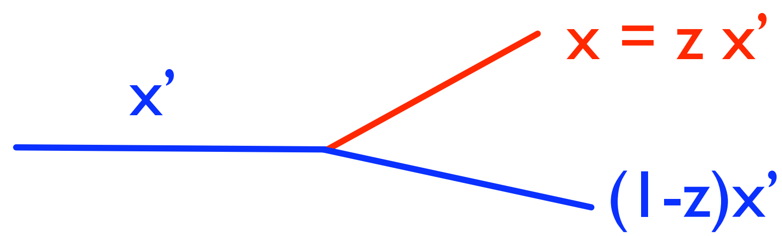
One incoming hard parton: $\sigma = \int dx f^{(P)}(x,\mu^2)\hat{\sigma}(xs,\mu^2)$

Two incoming hard partons: $\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1,\mu^2) f_2^{(P_2)}(x_2,\mu^2)\hat{\sigma}(x_1 x_2 s,\mu^2)$

Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces it



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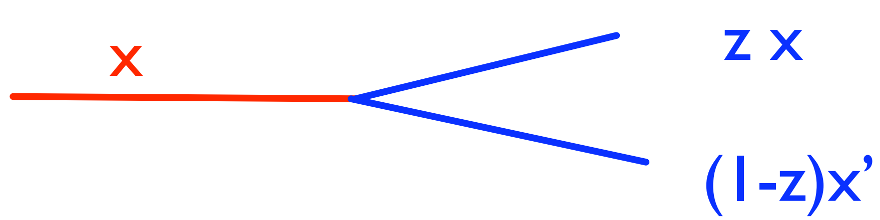

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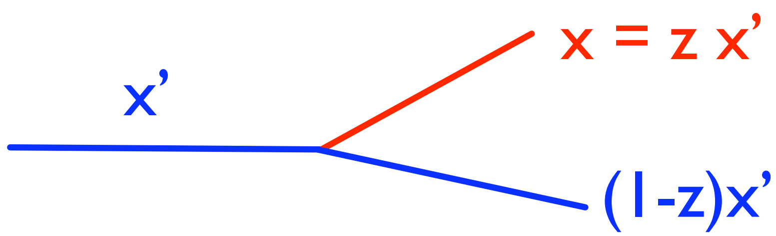

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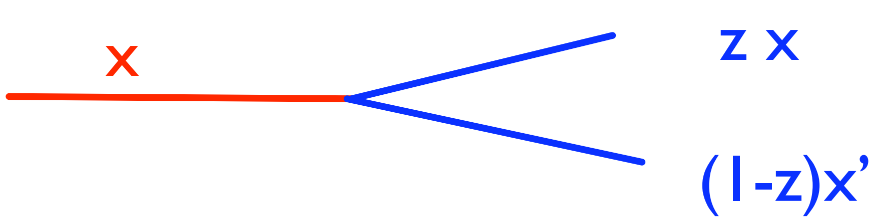


$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} \hat{P}(z) f(x',\mu^2) \delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} \hat{P}(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

$P(z) = \hat{P}(z)_+$

regulated splitting function

The plus prescription

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z) \left(g(z) - g(1)\right)$$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

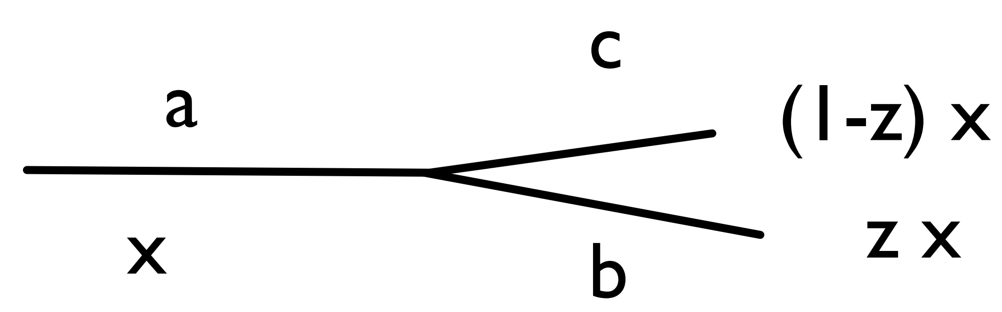
Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Conventions for splitting functions

There are various partons types. Standard notation:



$P_{ba}(z)$

Accounting for the different species of partons the DGLAP equations become:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z},\mu^2\right)$$

This is a system of coupled integro/differential equations

The above convolution in compact notation:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2)$$

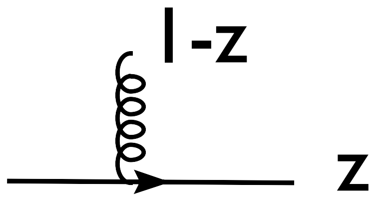
General DGLAP equation

Evolution equations in the general case:

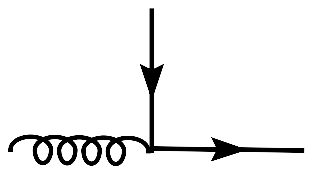
$$\mu^2 \frac{\partial f_i(z,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2) \qquad P_{ij}(x) = \frac{\alpha_s}{2\pi} P_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2)} + \dots$$

Leading order splitting functions:

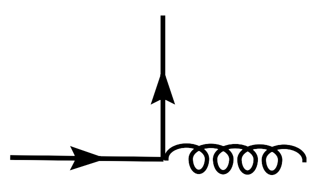
$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ + \frac{3}{2}\delta(1-z) \right]$$



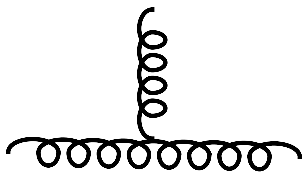
$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R \left(z^2 + (1-z) \right)$$



$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$



$$P_{gg}^{(0)} = 2C_A \left[z \left(\frac{1}{1-z} \right)_+ + \frac{1-z}{z} + z(1-z) + b_0 \delta(1-x) \right]$$



NB: at higher orders $P_{q_i q_j}$ arise

Properties of splitting functions

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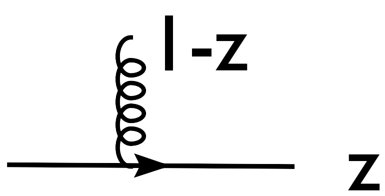

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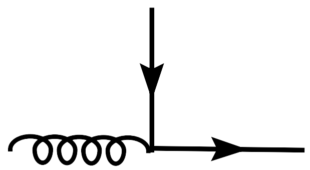

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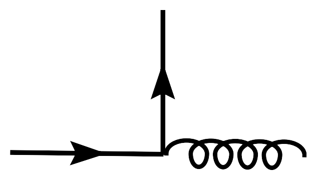
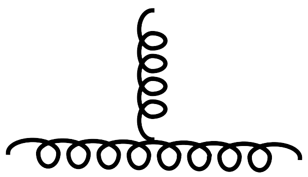


$$P_{gg}^{(0)} = 2C_A\left[z\left(\frac{1}{1-z}\right)_+ + \frac{1-z}{z} + z(1-z) + b_0\delta(1-z)\right]$$



- P_{qg} anf P_{gg} symmetric under z (1-z)
- P_{qq} and P_{gg} divergence for z=1 (soft gluon)
- P_{gq} and P_{gg} divergenge for z=0 (soft gluon)
- P_{qg} no soft divergence for gluon splitting to quarks

➠ gluon PDF grows at small x

History of splitting functions

- ☑ $P_{ab}^{(0)}$: Altarelly, Parisi; Gribov-Lipatov; Dokshitzer (1977)
- ☑ $P_{ab}^{(1)}$: Curci, Furmanski, Petronzio (1980)
- ☑ $P_{ab}^{(2)}$: Moch, Vermaseren, Vogt (2004)

☛ $P_{ab}^{(2)}$: hardest calculation ever performed in perturbative QCD

☛ Essential input for NNLO pdfs determination (state of the art today)

Singlet and non-singlet

The $2n_f+1$ evolution equations explicitly:

$$\mu^2 \frac{\partial q_i}{\partial \mu^2} = \sum_j P_{q_i q_j} \otimes q_j + P_{q_i g} \otimes g$$

$$\mu^2 \frac{\partial g}{\partial \mu^2} = \sum_j P_{g q_j} \otimes (q_j + \bar{q}_j) + P_{gg} \otimes g$$

Introduce the non-singlet and singlet combinations

$$\boxed{q^{\mathrm{NS}} = q_i - q_k} \qquad \boxed{\Sigma = \sum_{i=1}^{n_f} (q_i + \bar{q}_i)}$$

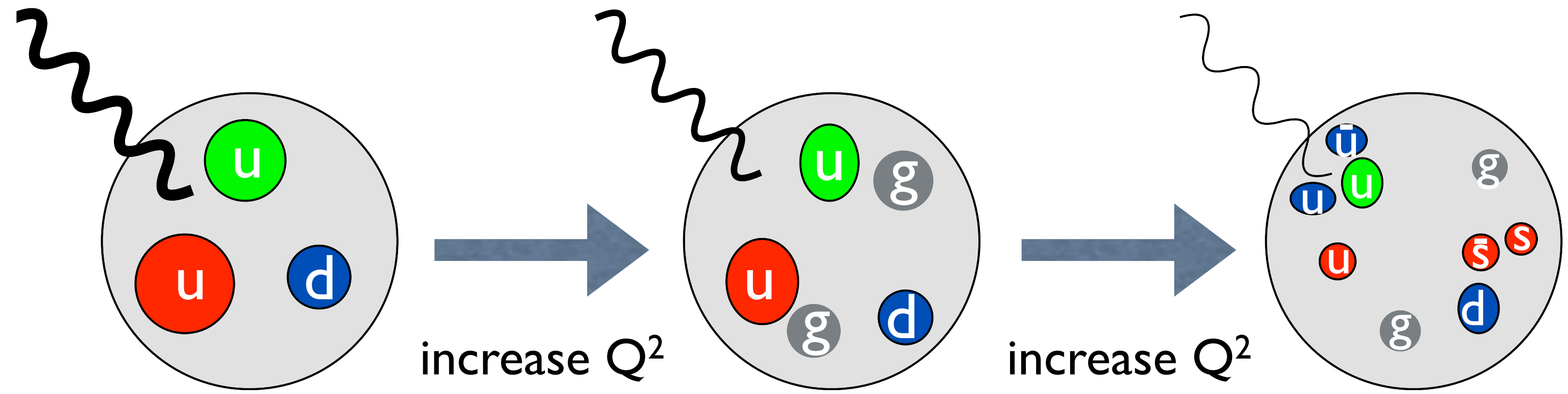
Then the non-singlet evolution decouples from the gluon, while the singlet and gluon evolve according to coupled equations

$$\mu^2 \frac{\partial q^{\mathrm{NS}}}{\partial \mu^2} = P_{qq} \otimes q^{\mathrm{NS}} \qquad \mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & Q_{qg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how can we access the gluon distribution: because of the coupled DGLAP evolution we can access the gluon distribution indirectly

DGLAP in Mellin space

How does one solve DGLAP equations?

One possibility: go to Mellin space

$$f_i(N, \mu^2) = \int_0^1 dx\, x^{N-1}\, f_i(x, \mu^2)$$

The advantage of Mellin transform: convolutions ⇒ ordinary products

Exercise: show that $(f \otimes g)(N) = f(N) g(N)$

The disadvantage of Mellin transform: need to evaluate inverse Mellin transform at the end

$$f_i(x, \mu^2) = \frac{1}{2\pi i} \int_C dN\, x^{-N}\, f_i(N, \mu^2)$$

Exercise: show that the above is indeed the inverse Mellin transform

Anomalous dimensions

Evolution equation for the non-singlet in Mellin space (for simplicity)

$$\mu^2 \frac{\partial q^{\mathrm{NS}}(N,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \gamma_{qq}(N, \alpha_s(\mu^2))\; q^{\mathrm{NS}}(N,\mu^2)$$

Where the anomalous dimension is given by

$$\gamma_{qq}(N,\alpha_s(\mu^2)) = \int_0^1 dx\, x^{N-1}\, P_{qq}(x,\alpha_s)$$

And similarly for the gluon and singlet component. At leading order:

$$\gamma_{qq}^{(0)} = C_F \left\{ -\frac{1}{2} + \frac{1}{N(N+1)} - 2\sum_{k=2}^{N} \frac{1}{k} \right\}$$

$$\gamma_{qg}^{(0)} = T_R \left\{ \frac{2+N+N^2}{N(N+1)(N+2)} \right\} \qquad \gamma_{qg}^{(0)} = C_F \left\{ \frac{2+N+N^2}{N(N^2-1)} \right\}$$

$$\gamma_{gg}^{(0)} = 2C_A \left\{ -\frac{1}{12} + \frac{1}{N(N-1)} + \frac{1}{(N+1)(N+2)} - \sum_{k=2}^{N} \frac{1}{k} \right\} - \frac{2}{3} n_f T_R$$

Solution in Mellin space

Given the anomalous dimension, the equation for non-singlet is

$$\mu^2 \frac{\partial q^{\rm NS}(N,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \gamma_{qq}(N, \alpha_s(\mu^2))\; q^{\rm NS}(N,\mu^2)$$

Solution in Mellin space

Given the anomalous dimension, the equation for non-singlet is

$$\mu^2 \frac{\partial q^{\rm NS}(N,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \gamma_{qq}(N,\alpha_s(\mu^2))\; q^{\rm NS}(N,\mu^2)$$

To lowest order one has

$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}} \qquad \gamma_{qq}(N,\alpha_s(\mu^2)) = \gamma_{qq}^{(0)}(N)$$

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Integrate the equation

$$q^{\rm NS}(N,Q^2) = q^{\rm NS}(N,Q_0^2) \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \qquad\qquad d^{(0)}(N) = \frac{\gamma^{(0)}(N)}{2\pi b_0}$$

Solution in Mellin space

Given the anomalous dimension, the equation for non-singlet is

$$\mu^2 \frac{\partial q^{\rm NS}(N,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\gamma_{qq}(N,\alpha_s(\mu^2))\; q^{\rm NS}(N,\mu^2)$$

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Integrate the equation

$$q^{\rm NS}(N,Q^2) = q^{\rm NS}(N,Q_0^2)\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \qquad\qquad d^{(0)}(N) = \frac{\gamma^{(0)}(N)}{2\pi b_0}$$

Finally need to take in inverse Mellin trasform to go back to x-space (usually this can be done only numerically)

Solution in x space

$$q^{\mathrm{NS}}(N,Q^2) = q^{\mathrm{NS}}(N,Q_0^2)\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \Rightarrow q^{\mathrm{NS}}(x,Q^2) = \frac{1}{2\pi i}\int_C dN x^{-N} q^{\mathrm{NS}}(N,Q^2)$$

Solution in x space

$$q^{\mathrm{NS}}(N,Q^2) = q^{\mathrm{NS}}(N,Q_0^2)\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \Longrightarrow q^{\mathrm{NS}}(x,Q^2) = \frac{1}{2\pi i}\int_C dN x^{-N} q^{\mathrm{NS}}(N,Q^2)$$

Explicit result shows that

$$d_{qq}^{(0)}(1) = 0 \qquad d_{qq}^{(0)}(N) < 0 \qquad N > 1$$

$$q^{\mathrm{NS}}(N,Q^2) < q^{\mathrm{NS}}(N,Q_0^2) \qquad Q^2 > Q_0^2 \qquad N > 1$$

Solution in x space

$$q^{\rm NS}(N,Q^2) = q^{\rm NS}(N,Q_0^2)\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \Rightarrow q^{\rm NS}(x,Q^2) = \frac{1}{2\pi i}\int_C dN x^{-N} q^{\rm NS}(N,Q^2)$$

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Large N ↔ small x (and viceversa)

Solution in x space

$$q^{\mathrm{NS}}(N,Q^2) = q^{\mathrm{NS}}(N,Q_0^2)\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \Rightarrow q^{\mathrm{NS}}(x,Q^2) = \frac{1}{2\pi i}\int_C dN x^{-N} q^{\mathrm{NS}}(N,Q^2)$$

Explicit result shows that

$$d_{qq}^{(0)}(1) = 0 \qquad d_{qq}^{(0)}(N) < 0 \qquad N > 1$$

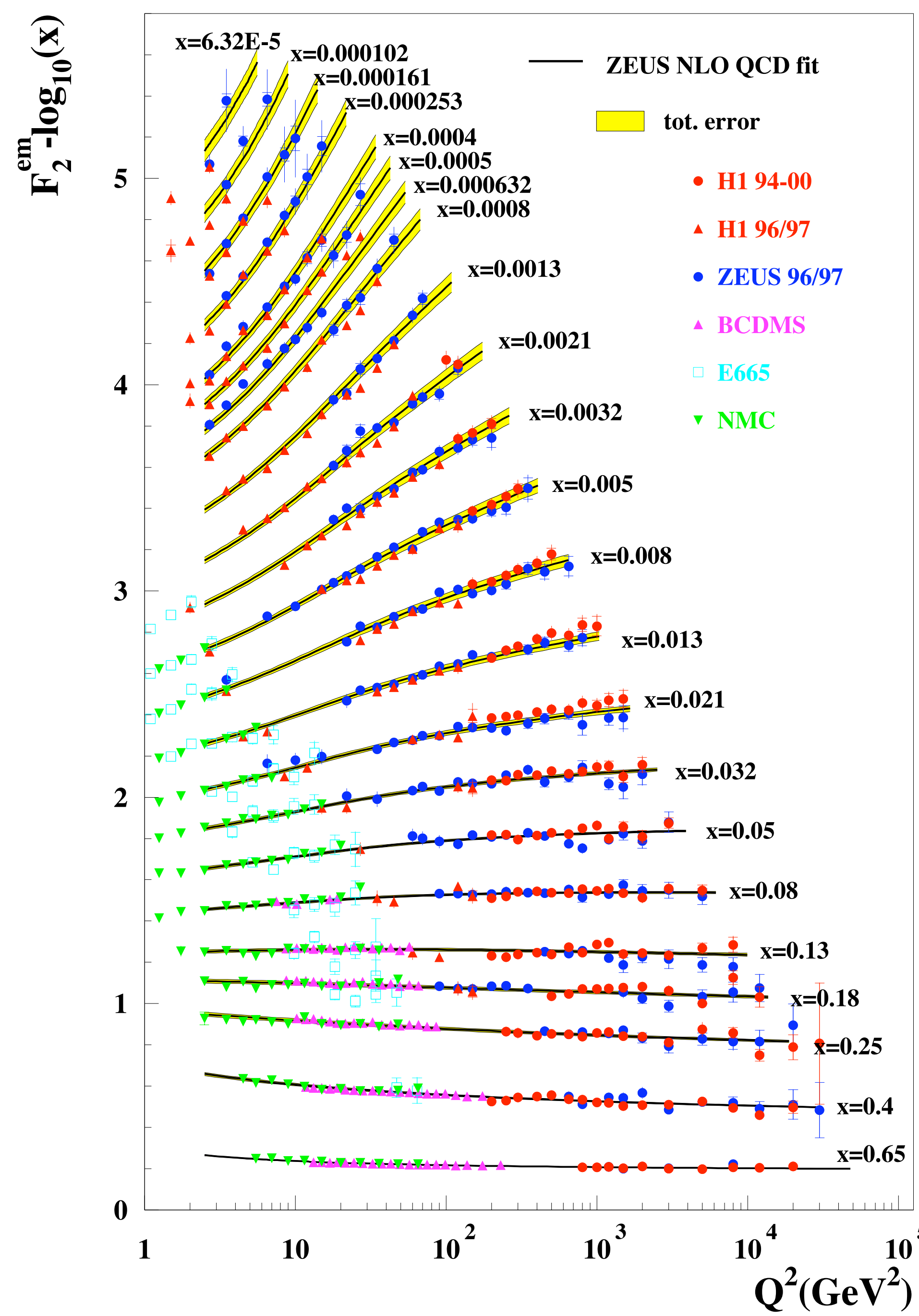
$$q^{\mathrm{NS}}(N,Q^2) < q^{\mathrm{NS}}(N,Q_0^2) \qquad Q^2 > Q_0^2 \qquad N > 1$$

Large N ↔ small x (and viceversa)

Increasing Q² $q^{NS}(x,Q^2)$ decreases at large x and increases at small x

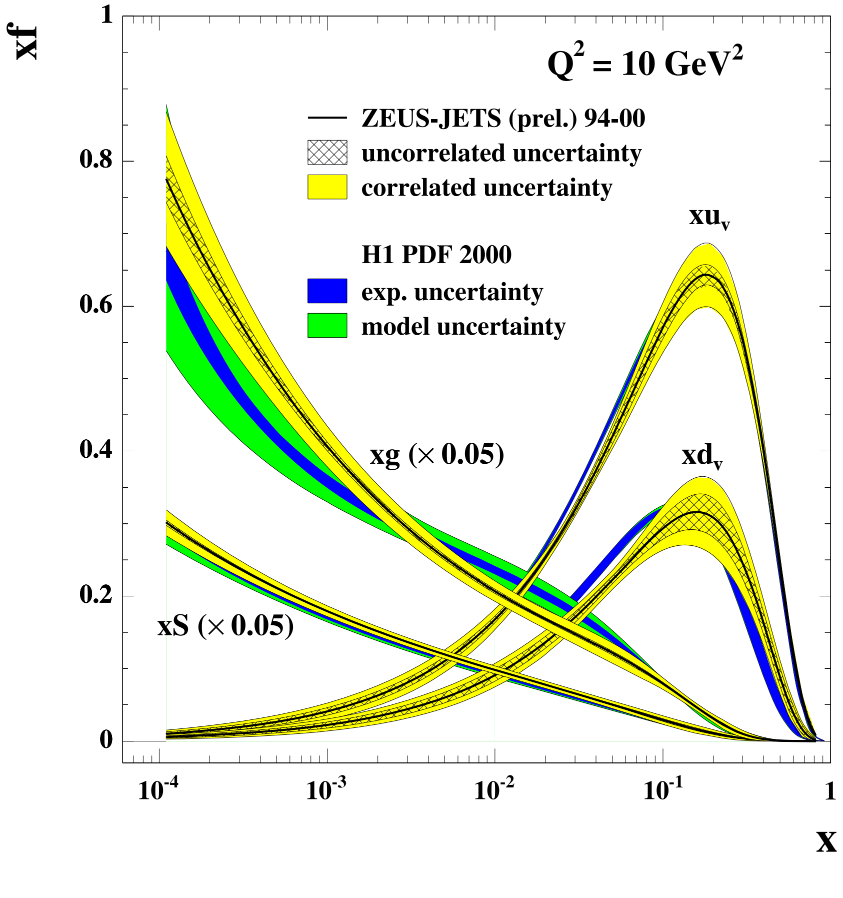
Physically: at larger x more phase space for gluon emission ⇒ reduction of quark momentum

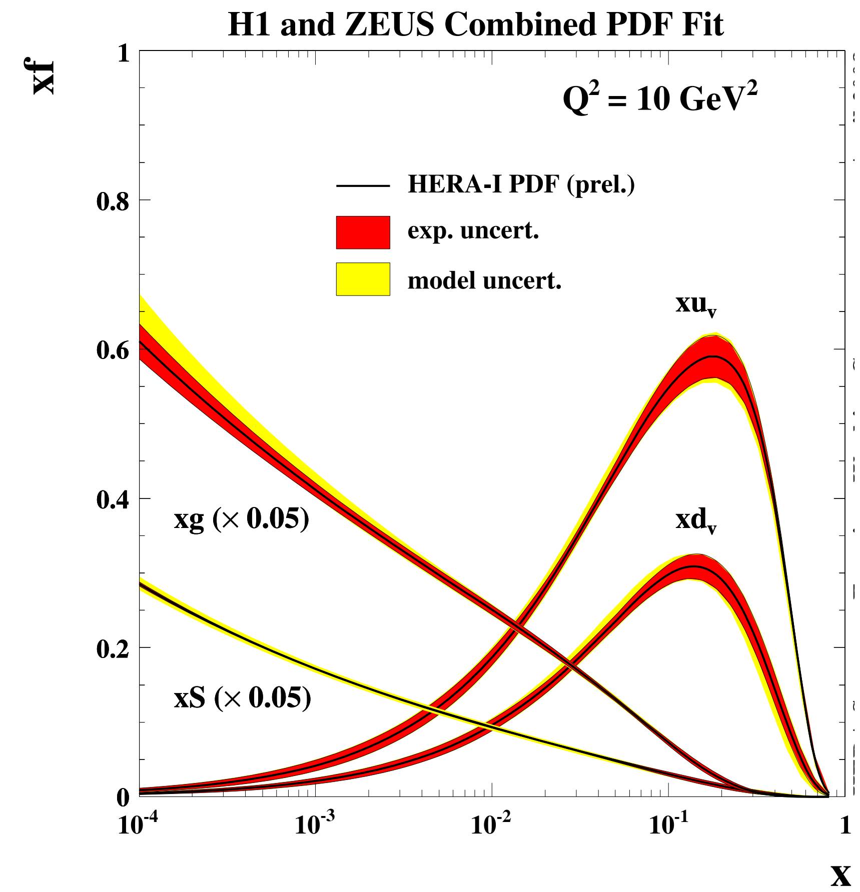
Data: F_2



Gluons crucial in driving the evolution

The Hera PDF





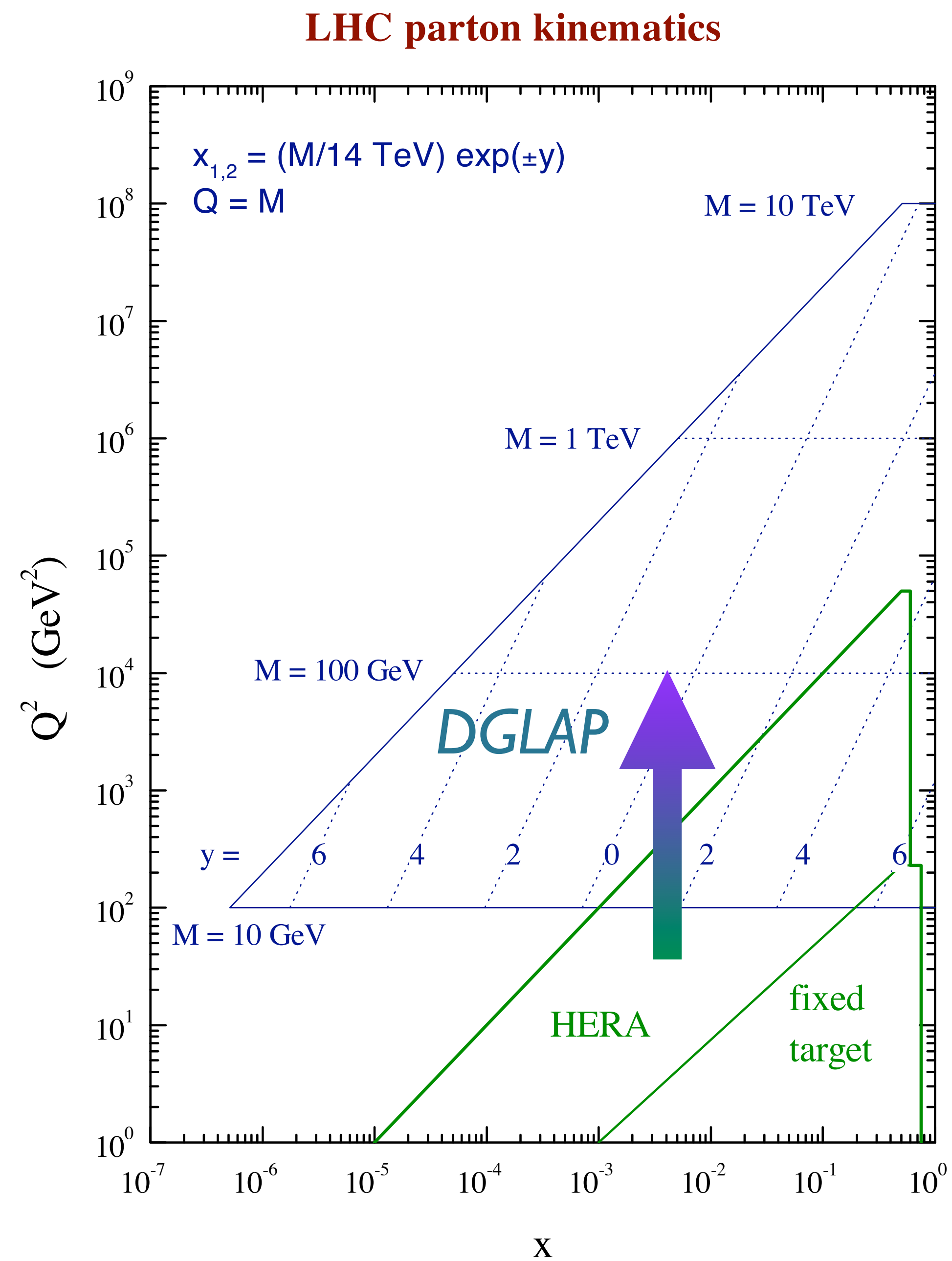
Hera structure function wg '08

Before: H1 and Zeus consistent within large uncertainties

Now: single Hera fit with improved error (still more data to come)

Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2-evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



PDF summary report, Hera-LHC '05

➠ *Hera: key and essential input to the LHC*

Parton densities: recent progress

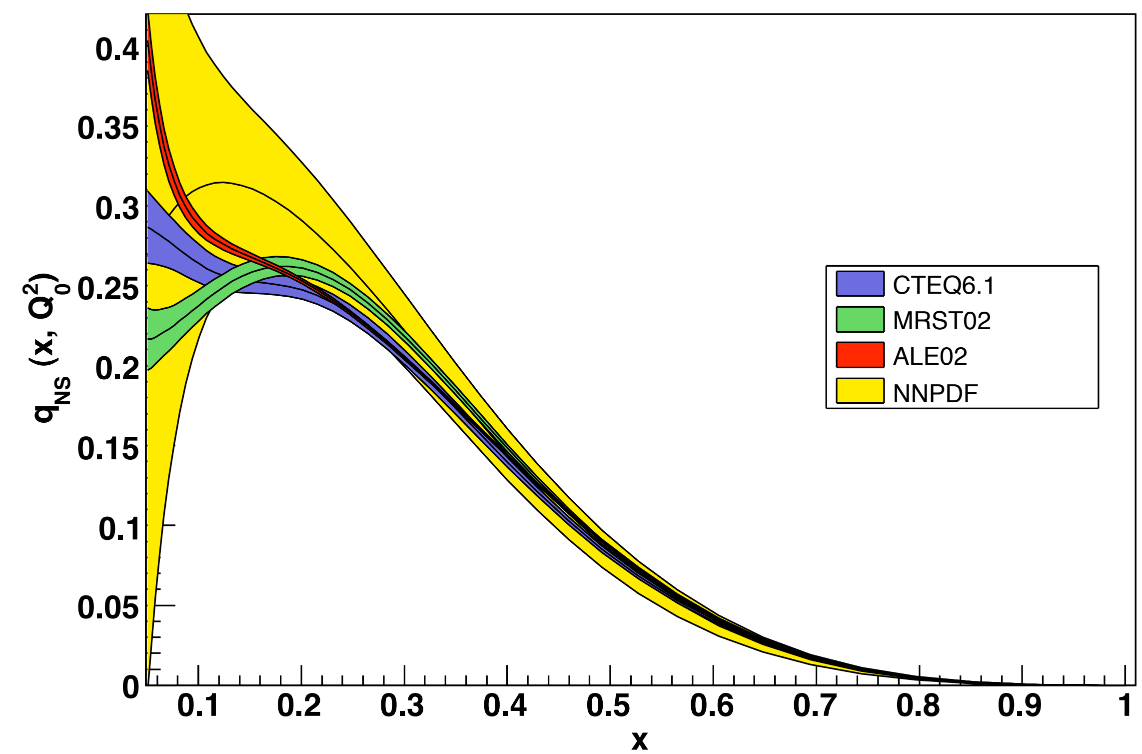
Recent major progress:

- full NNLO evolution (previous only approximate NNLO)
- full treatment of heavy flavors near the quark mass

  [Numerically: e.g. (6-7)% effect on Drell-Yan at LHC]
- more systematic use of uncertainties/correlations
- Neural Network PDFs [so far the nonsinglet case]

splitting functions at NNLO: Moch, Vermaseren, A. Vogt '04
[+ much related theory progress '04 -'08]
Alekhin, CTEQ, MSTW, NN collaborations

Parton densities: next?



Agreement not within quoted errors

- different values for $\alpha_s(M_Z)$
- differences in gluon distribution at small x and high x, and low-x sea quarks

➥ needs clarification

Open question:

- inclusion of more data in PDF fits

  e.g. neutrino DIS data from NuTeV, HERA jets, Tevatron high-E_T jets, new CDF lepton-asymmetry, new heavy flavour data from HERA, NuTeV dimuon ...

- or focus on selected (very clean) data ?

⇒ Description of PDFs reaching precision, but still some work ahead

Drell Yan processes

Drell-Yan processes: Z/W production (W → lν , Z → l^+l^-)

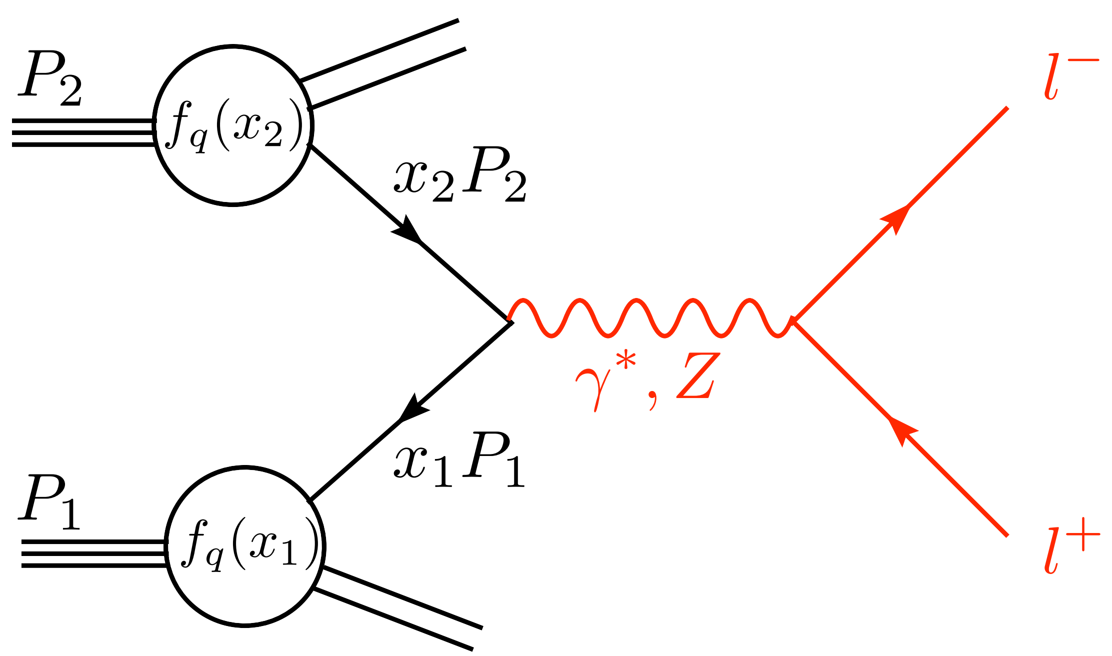
Very clean, golden-processes in QCD because

✓ dominated by quarks in the initial state

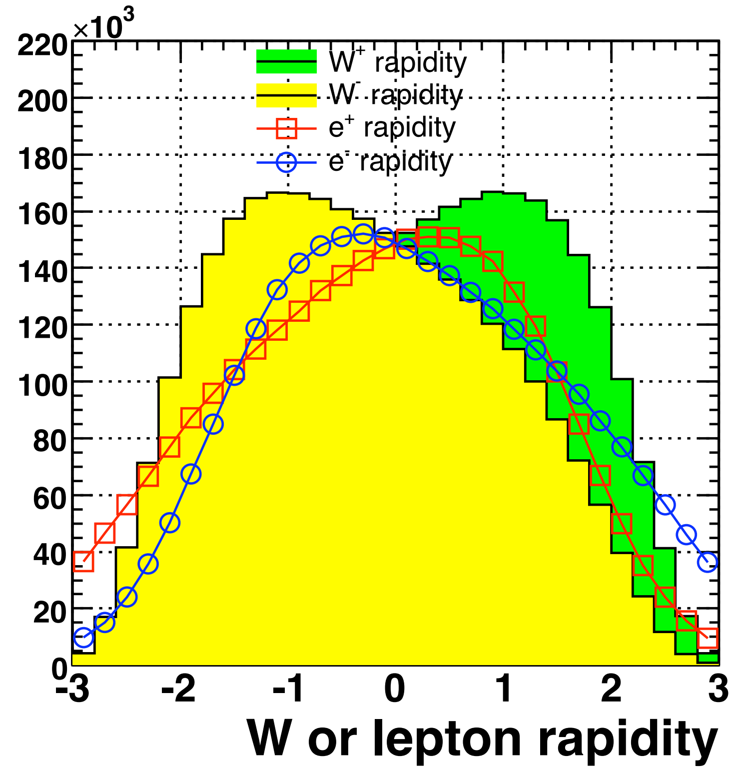
✓ no gluons or quarks in the final state (QCD corrections small)

✓ leptons easier experimentally (clear signature)

⇒ as clean as it gets at a hadron collider

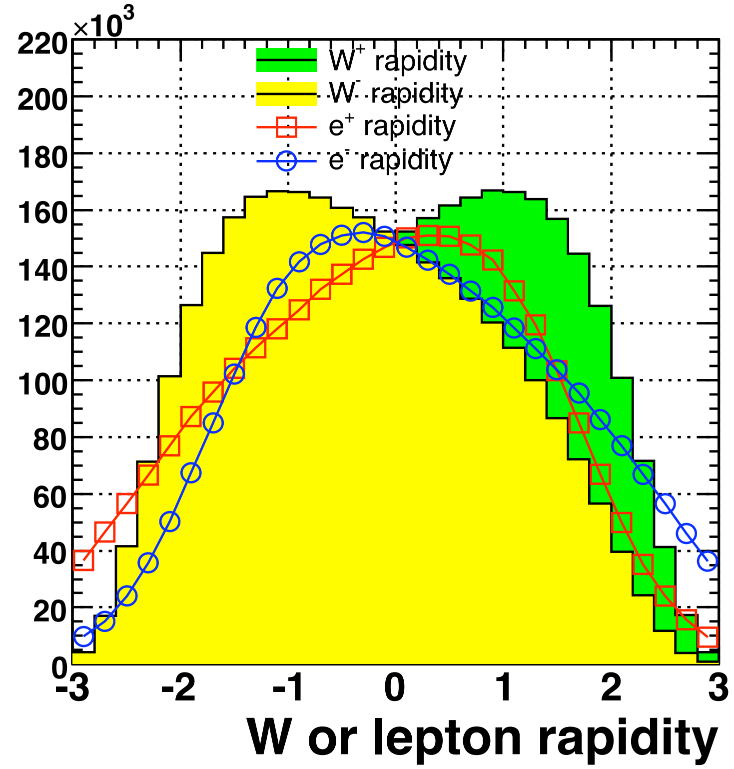


W asymmetry at the Tevatron



Question: where does the asymmetry come from ?

W asymmetry at the Tevatron



Question: where does the asymmetry come from ?

PDF effect: W^+ produced mainly from valance u-quark in the proton and valence $\bar{d}$ in the anti-proton. u-quarks are faster $\Rightarrow$ W^+ produced preferably in the proton direction (opposite for the W^-)

Rapidity asymmetry

Rapidity asymmetry: $A_W(y) = \dfrac{d\sigma_{W^+}/dy - d\sigma_{W^-}/dy}{d\sigma_{W^+}/dy + d\sigma_{W^-}/dy}$

Neglect sea quark contributions: $A_W(y) \sim \dfrac{u_p(x_1)\bar{d}_{\bar{p}}(x_2) - d_p(x_1)\bar{u}_{\bar{p}}(x_2)}{u_p(x_1)\bar{d}_{\bar{p}}(x_2) + d_p(x_1)\bar{u}_{\bar{p}}(x_2)}$

Use isospin symmetry $u_p(x) = \bar{u}_{\bar{P}}(x)$ etc. :

$$A_W(y) \sim \frac{u_p(x_1)d_p(x_2) - d_p(x_1)u_p(x_2)}{u_p(x_1)d_p(x_2) + d_p(x_1)u_p(x_2)}$$

Depends only on relative distribution $R(x) = \dfrac{d_p(x)}{u_p(x)}$

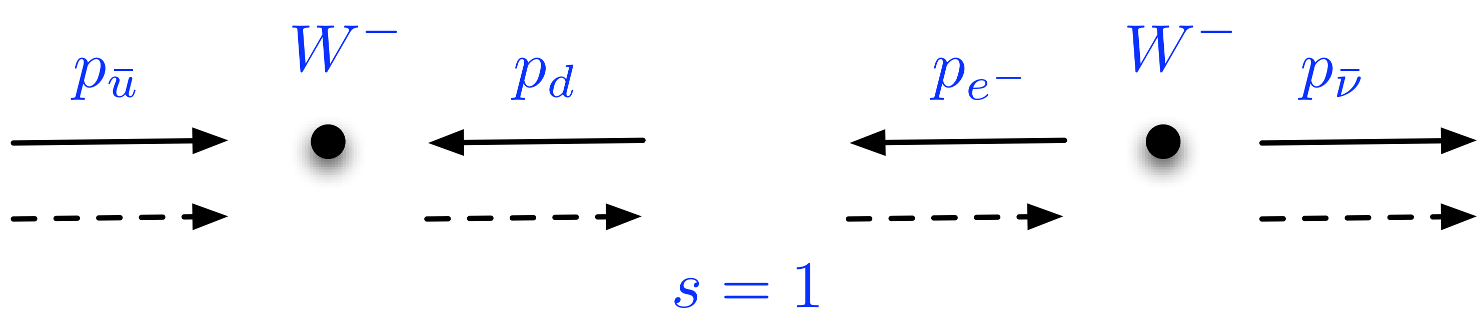
$$A_W(y) \sim \frac{R(x_2) - R(x_1)}{R(x_2) - R(x_1)}$$

$\Rightarrow$ very sensitive probe of relative shape of u and d distributions, but difficult because of neutrino in the final state

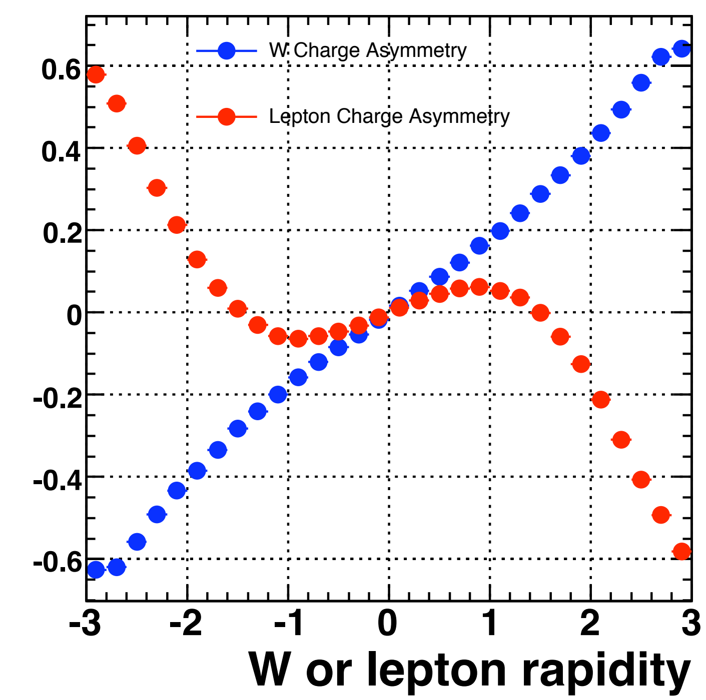
Charged lepton asymmetry

$$A_l(y) = \frac{d\sigma_{l^+}/dy - d\sigma_{l^-}/dy}{d\sigma_{l^+}/dy + d\sigma_{l^-}/dy}$$

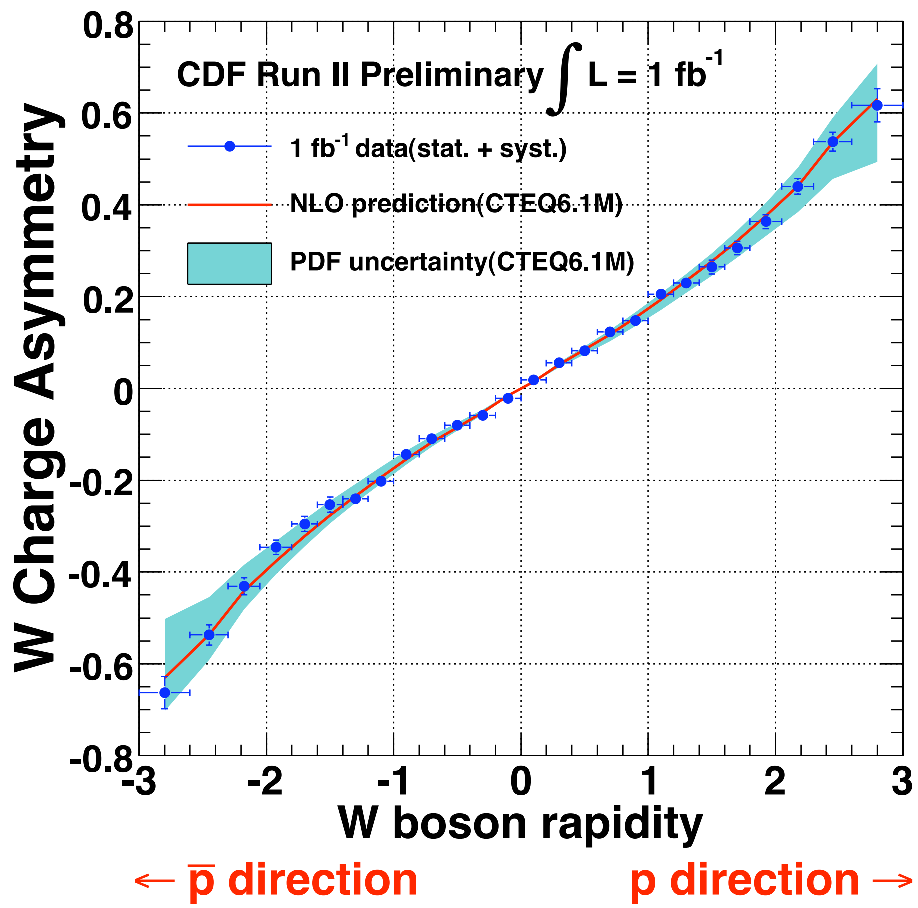
Complication: charged leptons not produced isotropically in W rest-frame: W couples to left (right)-handed fermions (anti-fermions)

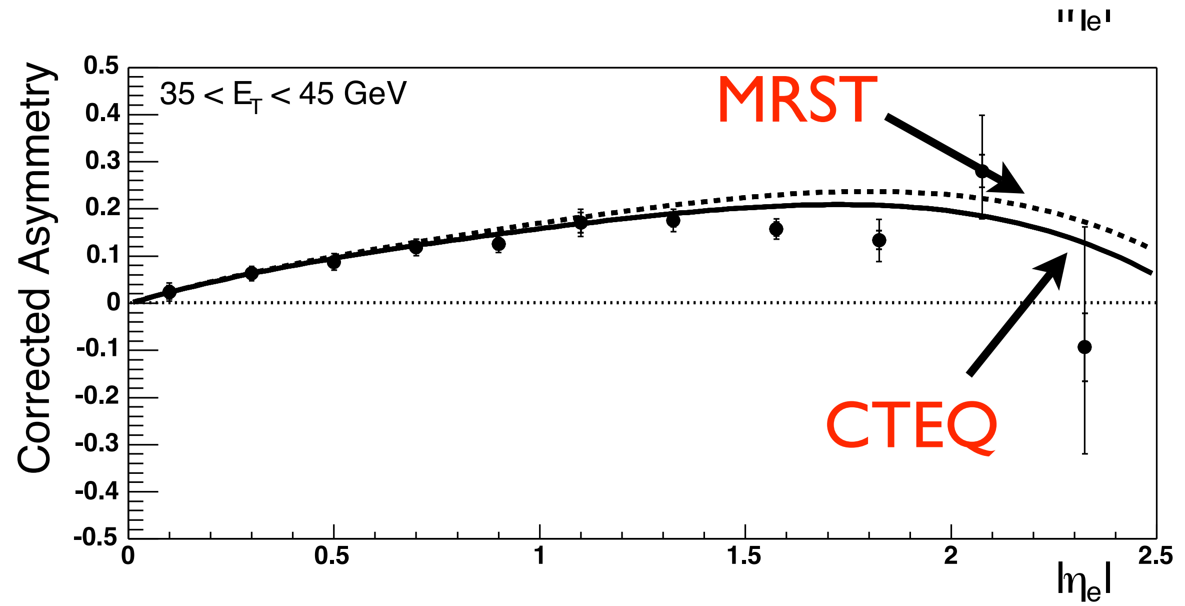


Outgoing fermion preferably in the direction of the incoming quark, which is mostly the proton direction (opposite as W asymmetry)



Asymmetry: CDF data





Recap.

- Parton model: incoherent sum of all partonic cross-sections
- Sum rules (momentum, charge, flavor conservation)
- Determination of parton densities (electron & neutrino scattering)
- Radiative corrections: failure of parton model
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ info on gluon
- Current status: NNLO + mass effects + new NN PDFs $\Rightarrow$ precision?
- Example: W^+W^- asymmetry at Tevatron